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Lorentz Violation and the Yukawa Potential

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Abstract

We analyze Lorentz violations in the bosonic sector of a Yukawa-type quantum field theory. The nonrelativistic potential may be determined to all orders in the Lorentz violation, and we find that only specific types of modifications to the normal Yukawa potential can be generated. The influence of this modified potential on scattering and bounds states is calculated. These results could be relevant to the search for new macroscopic forces, which may not necessarily be Lorentz invariant.

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There is presently quite a bit of interest in the possibility that Lorentz symmetry may not be exact in nature. If Lorentz violation does exist, it would be a very important clue regarding the nature of Planck-scale physics. There are a number of possible ways that Lorentz violation could arise, including spontaneous violations of the symmetry in string theory [1, 2] and elsewhere [3], mechanisms in loop quantum gravity [4, 5] and noncommutative geometry [6, 7], Lorentz violation through spacetime-varying couplings [8], and anomalous breaking of Lorentz and CPT symmetries [9].

To date, experimental tests of Lorentz violation have included studies of matter-antimatter asymmetries for trapped charged particles [10, 11, 12, 13] and bound state systems [14, 15], determinations of muon properties [16, 17], analyses of the behavior of spin-polarized matter [18, 19], frequency standard comparisons [20, 21, 22, 23], Michelson-Morley experiments with cryogenic resonators [24, 25], Doppler effect measurements [26, 27], measurements of neutral meson oscillations [28, 29, 30, 31], polarization measurements on the light from distant galaxies [32, 33, 34], and others. In order to evaluate the results of these experiments, it has been useful to develop a local quantum field theory that parameterizes the possible Lorentz violations. The most general such theory is the standard model extension (SME) [35, 36, 37]. The structure, as well as stability [38] and renormalizability [39], of this extension have been extensively studied. Most SME calculations are only done to first order in the Lorentz-violating coefficients, because practically, all Lorentz violations are small. Few results that are valid to all orders in the Lorentz violation are known, and new all-orders results, such as those we shall obtain here, are theoretically interesting.

Another area where high-precision experimental tests are important is in searching for new forces. Many of the same theories that predict Lorentz violation also predict that new forces may arise at, for example, submillimeter distances. Searches for weak, macroscopic forces other than electromagnetism and gravity have thus far not been successful. (For recent results, see [40, 41, 42]. There is, however, a hint of possible new forces acting in the Pioneer anomaly [43, 44].) Possible new interactions are frequently parameterized in terms of a Yukawa-type potential. While this kind of potential is sometimes chosen merely for its simplicity, it does have strong theoretical motivations, and it would be worthwhile to consider just what kinds of Yukawa-like interactions can exist within generalizations of the standard model. There are also Yukawa interactions in the standard model itself, both effective (as between pions and nucleons) and fundamental (involving the Higgs field). So it is reasonable to ask how Lorentz violation might affect Yukawa physics.

We shall examine this question using the effective field theory approach of the SME. First, we develop the nonrelativistic Yukawa theory, as modified by Lorentz violation. We pay particular attention to the potential energy, because this is what is generally constrained by precision macroscopic force experiments. Then we compare the Lorentz-violating Yukawa potential to the analogous quantity that arises when there is Lorentz violation in the electromagnetic sector.

We shall consider a theory with Lorentz violation in the bosonic sector only. The

Lagrange density is

$$\mathcal{L} = \bar{\psi}_A(i\partial\!\!\!/ - M)\psi_A + \frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) + \frac{1}{2}k^{\nu\mu}(\partial_\nu\phi)(\partial_\mu\phi) - \frac{1}{2}m^2\phi^2 - g\bar{\psi}_A\psi_A\phi. \tag{1}$$

A ϕ^4 interaction would also be required for renormalizability; however, we shall neglect it, as we shall only be considering tree-level phenomena. The index on the fermion field ψ_A labels distinguishable fermion species. These are included so that we may extract the interparticle potentials simply, without having to worry about exchange or annihilation scattering.

The coefficient $k^{\nu\mu}$ is the source of the Lorentz violation. $k^{\nu\mu}$ is a traceless symmetric tensor, which modifies the propagation of free scalar particles. This is the only superficially renormalizable form of Lorentz violation that can be written down involving only the real scalar field ϕ. If the elements of $k^{\nu\mu}$ have typical size m/M_P, where M_P is some large scale, then this theory is usable up to scales of order $\sqrt{mM_P}$. Above that scale, one encounters causality violations unless higher-derivative operators are included in the theory.

For a complex scalar field Φ, there is also a CPT-odd Lorentz-violating interaction, which has Lagrange density

$$\mathcal{L}_a = \left(\partial^\mu\Phi^*\right)\left(\partial_\mu\Phi\right) + ia^\mu\left[\Phi^*\left(\partial_\mu\Phi\right) - \left(\partial_\mu\Phi^*\right)\Phi\right] - m^2\Phi^*\Phi. \tag{2}$$

However, the current term multiplying a degenerates to zero in the real case, and it is not natural to consider a complex scalar field in connection with a Yukawa-type interaction, because of the theory's Hermiticity and symmetry structure. A theory with a multiplet of $2N$ real scalar fields can also be written in terms of N complex scalar fields, but an a-like term will violate the $SO(2N)$ symmetry.

In (1), we are neglecting Lorentz violations in the fermion sector, which are generally much more complicated. Because of the fermions' Dirac matrix structure, there are many more Lorentz-violating interactions possible. However, it is important to keep in mind that we are treating the most general possible bosonic Lorentz violation in this theory. Since any new long-range Yukawa interactions are known to be quite weak, the Lorentz violation associated with them does not have to be extremely strongly suppressed to have escaped detection. On the other hand, Lorentz violations for the most commonly observed fermions—electrons, muons, protons, and neutrons—have been relatively well constrained.

Moreover, the simple fact that we shall want to calculate a potential for this theory is actually another reason to consider Lorentz violation in the Yukawa boson sector only. Without rotation and Galilean invariance, the conventional definition of the nonrelativistic potential may not be useful, as Lorentz violations can affect the division of the effective Hamiltonian into potential and kinetic components in a nontrivial fashion. This fact can be made evident most easily by looking at the relativistic bosonic system described by $\mathcal{L}_a$. The action in this theory is a bilinear function of Φ^* and Φ, and so the theory is free.

In particular, a field redefinition

$$\Phi \rightarrow e^{ia\cdot x}\Phi,\ \Phi^* \rightarrow e^{-ia\cdot x}\Phi^* \tag{3}$$

converts the Lagrange density into

$$\mathcal{L}'_a = (\partial^\mu\Phi^*)\,(\partial_\mu\Phi) - (m^2 + a^2)\Phi^*\Phi. \tag{4}$$

So even though a couples to an expression containing derivatives, it can be seen as affecting the potential part of the Lagrangian. Normally, a^2 would be considered a small correction to the mass term, but for large values of a^2, the nature of the theory can change qualitatively. There exists a physical spectrum only for $a^2 \geq -m^2$, for if $a^2 < -m^2$, the energy is not bounded below. While this bosonic example is the simplest, similar considerations apply to fermions and in the nonrelativistic regime. When there is Lorentz violation, particle energies possess unorthodox structures, and so their division into kinetic and potential parts can be tricky and is not necessarily unambiguous. However, these problems with defining an interparticle potential do not exist if there is no Lorentz violation in the Lagrangian for the external particles in which we are interested. (The complexities of the energy-momentum relations in Lorentz-violating fermion theories are discussed further in [35, 45, 46], where particular attention is paid to the intimately related question of how the velocity behaves in these theories.)

The greatest virtue of the simple theory defined by (1) is that a number of tree-level results may be worked out exactly (i.e. to all orders in $k^{\nu\mu}$). The Yukawa potential will illustrate this. This potential will naturally be modified by the Lorentz violation, and we shall find that it is a relatively simple matter to determine the structure of these modifications nonperturbatively in $k^{\nu\mu}$.

To determine the potential, we consider the nonrelativistic scattering of two distinguishable fermions by one-ϕ exchange. The particles have initial (final) momenta p and k (p' and k'), and nonrelativistically these must take the form

$$\begin{array}{rr} p = (M, \vec{p}\,)\,, & k = \left(M, \vec{k}\right), \\ p' = (M, \vec{p}\,')\,, & k' = \left(M, \vec{k}'\right). \end{array} \tag{5}$$

So the momentum exchanged is

$$p' - p = q = (0, \vec{p}\,' - \vec{p}\,)\,, \tag{6}$$

up to corrections suppressed by powers of $|\vec{p}\,|/M$. Since we are considering a nonrelativistic system, only the k_{jl} portion of $k^{\nu\mu}$ (which breaks rotational invariance) will contribute. Any effects of k_{0j} are also suppressed by factors of $|\vec{p}\,|/M$.

The matrix element for the scattering process is then

$$i\mathcal{M} = \frac{-ig^2}{q^\mu q_\mu + k^{\nu\mu}q_\nu q_\mu - m^2}\left(2M\delta^{ss'}\right)\left(2M\delta^{rr'}\right), \tag{7}$$

where s and r (s' and r') label the incoming (outgoing) spins. The key quantity in this expression is the part that comes from the Yukawa boson propagator. This is essentially the three-dimensional Fourier transform of the modified potential:

$$\tilde{V}(\vec{q}) = \frac{-g^2}{q_j q_j - k_{jl} q_j q_l + m^2}. \tag{8}$$

To get the interparticle potential, we simply invert the Fourier transform,

$$V(\vec{r}) = \int \frac{d^3 q}{(2\pi)^3} \frac{-g^2}{q_j q_j - k_{jl} q_j q_l + m^2} e^{i q_j r_j}. \tag{9}$$

We can reduce this integral to the one that appears in the Lorentz-invariant case by making changes of variables. First, let us define a matrix K with elements $K_{jl} = \delta_{jl} - k_{jl}$. This matrix is symmetric and, presuming $|k_{jl}| \ll 1$, positive definite, so by making an orthogonal rotation of the coordinates, we may reduce it to a diagonal matrix. Then in the rotated coordinates, the denominator in the Fourier transform expression becomes $K_{11} q_1^2 + K_{22} q_2^2 + K_{33} q_3^2 + m^2$. We may then rescale the integration variables by introducing $\bar{q}_j = \sqrt{K_{jj}} q_j$. (Here and for the rest of this paragraph, there will be no implied summations over the index j; other repeated Roman indices are still summed, however.) The exponent becomes $e^{i \bar{q}_l \bar{r}_l}$, with $\bar{r}_j = r_j / \sqrt{K_{jj}}$, and the integration measure transforms as

$$d^3 q = \frac{d^3 \bar{q}}{\sqrt{\det K}}. \tag{10}$$

This brings $V(\vec{r})$ into the form

$$V(\vec{r}) = \frac{1}{\sqrt{\det K}} \int \frac{d^3 \bar{q}}{(2\pi)^3} \frac{-g^2}{\bar{q}_l \bar{q}_l + m^2} e^{i \bar{q}_l \bar{r}_l}, \tag{11}$$

and the calculation is then elementary:

$$V(\vec{r}) = -\frac{g^2}{4\pi \sqrt{\det K}} \frac{e^{-m\bar{r}}}{\bar{r}}, \tag{12}$$

where

$$\bar{r} = \sqrt{\bar{r}_l \bar{r}_l} \tag{13}$$

$$= \sqrt{\left(K^{-1}\right)_{ln} r_l r_n}. \tag{14}$$

Since (12) and (14) are written in terms of the basis-independent quantities $\det K$ and $\left(K^{-1}\right)_{jl} r_j r_l$ (now summing over j again), these expressions are valid in any basis. In particular, they hold in the original basis in which K_{jl} is not necessarily diagonal. So they give an expression for the Lorentz-violation-modified Yukawa potential which is correct to all orders in k_{jl}.

As a simple check of the correctness of this result, we may consider the case in which $k^{\nu\mu} = Cg^{\nu\mu}$. Then there is no physical Lorentz violation, and $k^{\nu\mu}$ may be eliminated by rescaling the field ϕ. The net result is a rescaling of g and m each by a factor of $1/\sqrt{1+C}$, which is just what (12) and (14) together indicate in this instance.

Obviously, it is useful to expand the expression for $V(\vec{r})$ to leading order in k_{jl}. The determinant part is easy to expand. To leading order, $\det K$ is equal to $1 - k_{jj}$, so

$$\frac{1}{\sqrt{\det K}} \approx 1 + \frac{1}{2}k_{jj}, \tag{15}$$

and this depends only on the trace k_{jj} over the space part of $k^{\nu\mu}$. However, this dependence on the trace can simply be absorbed into the overall scaling of g. The expansion of $\bar{r}$ to leading order in k_{jl} is also simple. Since $\left(K^{-1}\right)_{jl} \approx \delta_{jl} + k_{jl}$, we have

$$\bar{r} \approx r\left(1 + \frac{1}{2}k_{jl}\hat{r}_j\hat{r}_l\right), \tag{16}$$

where $\hat{r}$ is the unit vector $\vec{r}/r$ in the direction of $\vec{r}$. So the complete $\mathcal{O}(k^{\nu\mu})$ expression for $V(\vec{r})$ is

$$V(\vec{r}) \approx -\frac{g^2}{4\pi}\frac{e^{-mr}}{r}\left[1 + \frac{1}{2}k_{jj} - \frac{1}{2}\left(1+mr\right)k_{jl}\hat{r}_j\hat{r}_l\right]. \tag{17}$$

Since $\vec{r}$ is the only vector appearing in the problem that can be contracted with k_{jk}, the expressions k_{jj} and $k_{jl}\hat{r}_j\hat{r}_l$ are really the only tensor structures that can arise at this order. As previously noted, the first of these quantities only scales the interaction strength (as long as we consider effects only in a fixed reference frame); however, the second quantity enters in a particular and interesting fashion. At small distances (less than the boson Compton wavelength), the constant term in $(1+mr)$ dominates, but at larger distances, the mr part gives the dominant Lorentz-violating contribution unless the scalar field is massless.

The particle-antiparticle potential is the same. This follows from the fact that the fermionic structure in the theory is completely conventional. So we have shown that (12) is essentially the only interparticle potential that can arise from a renormalizable Lorentz-violating modification of the Yukawa sector. At leading order, rotation invariance is broken only by a term of the form $k_{jl}\hat{r}_j\hat{r}_l$, while purely phenomenalistic modifications of the Yukawa potential could also involve the structure $v_j\hat{r}_j$ (for some constant vector $\vec{v}$) or similar contractions with tensors of more than two indices.

We can also easily derive the nonrelativistic scattering cross-section from our expression for $i\mathcal{M}$. In the center of mass frame, with the two initial particles approaching one-another along the z-axis, each with (nonrelativistic) energy ϵ, the momentum exchange is just

$$\vec{q} = \sqrt{2M\epsilon}\left(\cos\varphi\sin\theta, \sin\varphi\sin\theta, \cos\theta - 1\right) = 2\sqrt{2M\epsilon}\left|\sin\left(\theta/2\right)\right|\hat{q}, \tag{18}$$

in terms of the scattering angles θ and ϕ. So the cross section for scattering with the particle spins unchanged is

$$\frac{d\sigma}{d\Omega}=\frac{g^2}{256\pi^2\epsilon^2\left[4\sin^2\left(\theta/2\right)\left(1-k_{jl}\hat{q}_j\hat{q}_l\right)+\frac{m^2}{2M\epsilon}\right]^2}, \tag{19}$$

and to leading order in $k^{\nu\mu}$, this is just

$$\frac{d\sigma}{d\Omega}\approx\frac{g^2}{256\pi^2\epsilon^2\left[4\sin^2\left(\theta/2\right)+\frac{m^2}{2M\epsilon}\right]^2}\left[1+\frac{8\sin^2\left(\theta/2\right)k_{jl}\hat{q}_j\hat{q}_l}{4\sin^2\left(\theta/2\right)+\frac{m^2}{2M\epsilon}}\right]. \tag{20}$$

Since for small enough m, the Yukawa potential has bound states, we can also see how these are affected by the Lorentz violation. Our results still apply in the $m=0$ limit, and in this limit, we can extract the leading-order energy shifts for the nonrelativistic bound states by ordinary perturbation theory. However, because of the degeneracies of the hydrogenic spectrum, there is a secular matrix to diagonalize, and this cannot generally be done in closed form. The energy shift has a simple form only for the ground state, where it is equal to

$$E=-\frac{M}{2}\left(\frac{g^2}{4\pi}\right)^2\left(1+\frac{2}{3}k_{jj}\right). \tag{21}$$

The potential that we have found is similar in structure to the modified electrostatic potential A^0 that appears in a CPT-even Lorentz-violating electromagnetic theory. In such a theory, the Lorentz violation arises from a Lagrange density

$$\mathcal{L}_F=-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-\frac{1}{4}k_F^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}-A^\mu j_\mu. \tag{22}$$

The four-vector potential for a point charge q (appropriately normalized) in this theory is [47]

$$A^0(\vec{r}) \approx \frac{q}{4\pi}\frac{1}{r}\left[1-\left(k_F\right)_{0j0l}\hat{r}_j\hat{r}_l\right], \tag{23}$$

$$A_j(\vec{r}) \approx \frac{q}{4\pi}\frac{1}{r}\left[\left(k_F\right)_{0ljl}-\left(k_F\right)_{jl0n}\hat{r}_l\hat{r}_n\right]. \tag{24}$$

The angular dependences of $A^0(\vec{r})$ and $V(\vec{r})$ are essentially the same, but what distinguishes the vector theory is the presence of a nonzero $\vec{A}$ (and correspondingly, a nonzero $\vec{B}$), even in the absence of moving charges. This mixing of electrostatic and magnetostatic effects gives us the possibility of distinguishing, via magnetic measurements, between scalar- and vector-mediated Lorentz-violating forces. The specific Lorentz-violating coefficient $\left(k_F\right)_{0j0l}$ that appears in (23) can actually be (to leading order) exported to the matter sector by a redefinition of the coordinates [47]. This results in a proportional k_{jl}

coefficient for each of the matter fields, but this differs from the Yukawa case we have considered, because the fermions will also have the same k_{jl} as the scalar bosons.

If, in the future, weak new forces are discovered, then it is definitely an interesting question whether these forces are Lorentz invariant. We have shown that a Lorentz-violating, renormalizable effective field theory can support only a limited class of modifications to the nonrelativistic Yukawa potential. Lorentz violations in the bosonic sector can only result in the specific CPT-even changes to the potential that are described by (12). This expression for $V(\vec{r})$ is correct to all orders in the Lorentz violation, but at leading order it closely resembles the electrostatic potential in a Lorentz-violating electromagnetic theory. However, the electromagnetic theory will generally also contain mixing between electric and magnetic interactions, which distinguishes it from the scalar Yukawa theory. Identifying the field-theoretical origins of any experimentally observed Lorentz violations would be very important, and these results provide the tools to help make such an identification.

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